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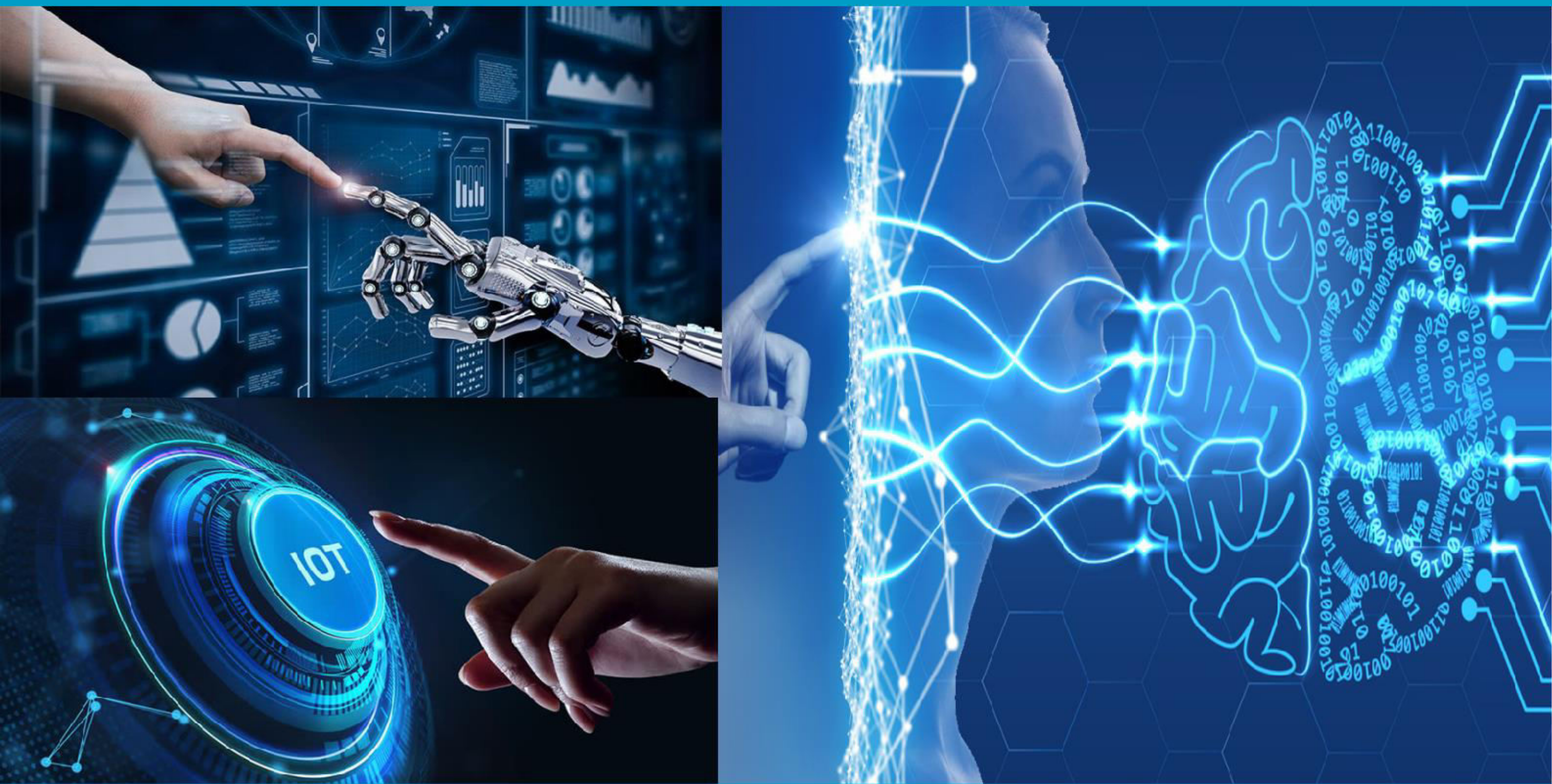

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Contribution of Indian Mathematician to World Mathematics from Upanishad Era: An Analysis

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ABSTRACT: Mathematics has played a significant role in the development of Indian culture for millennia. Mathematical ideas that originated in the Indian subcontinent have had a profound impact on the world.It should come as no surprise that the first recorded use of the number zero, recently discovered to be made as early as the 3rd or 4th century, happened in India. Mathematics on the Indian subcontinent has a rich history going back over 3,000 years and thrived for centuries before similar advances were made in Europe, with its influence meanwhile spreading to China and the Middle East.As well as giving us the concept of zero, Indian mathematicians made seminal contributions to the study of Trigonometry, Algebra, Arithmetic and negative numbers among other areas. Perhaps most significantly, the decimal system that we still employ worldwide today was first seen in India.Whereas Vedic Mathematics is a collection of Methods or Sutras to solve numerical computations quickly and faster. It consists of 16 Sutras called Formulae and 13 sub-sutras called Sub Formulae, which can be applied in solving of problems ofArithmetic, Algebra, Geometry, Calculus, conics, etc.

KEYWORDS: Mathematics, mathematician, Vedic, Algebra, sutra.

I.INTRODUCTION

Indian Mathematics emerged in the Indian subcontinent from 1200 BCE until the end of the 18th century. In the classical period of Indian Mathematics (400 CE to 1200 CE), significant contributions were made by scholars like Aryabhata, Brahmagupta, Bhaskara II and Varahamihira.

The decimal number system in use today was first recorded in Indian Mathematics. Indian mathematicians made early contributions to the study of the concept of zero as a number,negative numbers,Arithmeticand Algebra. In addition to these,Trigonometry was further advanced in India and suggestively, the modern definitions of sine and cosine were developed at that time. These mathematical concepts were transmitted to the Middle East, China and Europe and led further developments which now form the foundations of many areas of Mathematics.

Ancient and medieval Indian mathematical works were all composed in Sanskrit. Those theories usually consisted of a section of Sutras in which a set of rules or problems were stated with great economy in order to aid memorization by a student. This was followed by a second section consisting of a prose commentary that explained the problem in more detail and provided justification for the solution. In the prose section, the form was not considered so important as the ideas involved.All mathematical works were orally transmitted until approximately 500 BCE, thereafter, they were transmitted both orally and in manuscript form. The oldest extant mathematical document produced on the Indian subcontinent is the cane bark Bakhshali Manuscript, discovered in 1881 in the village of Bakhshali, near Peshawar and is likely from the 7th century CE.

A later landmark in Indian mathematics was the development of the series expansions for Trigonometric Functions (sine, cosine, and arc tangent) by mathematicians of the Kerala school in the 15th century CE. Their remarkable work, completed two centuries before the invention of Calculus in Europe, provided what is now considered the first example of a Power Series (apart from geometric series).However, they did not formulate a systematic theory of Differentiation and Integration, nor is there any direct evidence of their results being transmitted outside Kerala.

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II. PREHISTORY OF THE RELATED LITERATURE

Excavations at Harappa, Mohenjo-daro and other sites of the Indus Valley civilisation have uncovered evidence of the use of practical Mathematics. The people of the Indus Valley Civilization manufactured bricks whose dimensions were in the proportion 4:2:1, considered favourable for the stability of a brick structure. They used a standardised system of weights based on the ratios: 1/20, 1/10, 1/5, 1/2, 1, 2, 5, 10, 20, 50, 100, 200 and 500 with the unit weight equalling approximately 28 grams and nearly equal to the English ounce or Greek uncia. They mass-produced weights in regular geometrical shapes, which included hexahedra, barrels, cones, and cylinders, thereby demonstrating knowledge of basic geometry.

The inhabitants of Indus civilisation also tried to standardise measurement of length to a high degree of accuracy. The Mohenjo-daro ruler—whose unit of length (approximately 1.32 inches or 3.4 centimetres) was divided into ten equal parts. Bricks manufactured in ancient Mohenjo-daro often had dimensions that were integral multiples of this unit of length.

Hollow cylindrical objects made of shell and found at Lothal (2200 BCE) and Dholavira are demonstrated to have the ability to measure angles in a plane, as well as to determine the position of stars for navigation.

The religious texts of the Vedic Period provide evidence for the use of large numbers.

The solution to partial fraction was known to the Rigvedic People as states in the purushSukta. The Satapatha Brahmana (7th century BCE) contains rules for ritual geometric constructions that are similar to the Sulba Sutras.

The Sulba Sutras, literally which means ‘Aphorisms of the Chords’ in Vedic Sanskrit (700–400 BCE) list rules for the construction of sacrificial fire altars. Most mathematical problems considered in the Sulba Sutras spring from a single theological requirement that of constructing fire altars which have different shapes but occupy the same area. The altars were required to be constructed of five layers of burnt bricks with the further condition that each layer consist of 200 bricks and that no two adjacent layers have congruent arrangements of bricks.

According to Hayashi 2005, p. 363, the Sulba Sutras contain the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians.

Among the scholars of the post-Vedic period who contributed to mathematics, the most notable is Pingala (300–200 BCE), a music theorist who authored the ChhandasShastra, a Sanskrit treatise on prosody. There is evidence that in his work on the enumeration of syllabic combinations. Pingala stumbled upon both Pascal's triangle and Binomial Coefficients, although he did not have knowledge of the Binomial Theorem itself. Pingala's work also contains the basic ideas of Fibonacci numbers.

Katyayana (3rd century BCE) is notable for being the last of the Vedic mathematicians. He wrote the KatyayanaSulba Sutra, which presented much Geometry, including the general Pythagorean Theorem and a computation of the square root of 2 correct to five decimal places.

III.ORAL TRADITION

Mathematicians of ancient and early medieval India were almost all Sanskritpandits who were trained in Sanskrit language and literature, and possessed a common stock of knowledge in grammar, exegesis and logic. Memorization of what is heard (sruti in Sanskrit) through recitation played a major role in the transmission of sacred texts in ancient India. Memorisation and recitation were also used to transmit philosophical and literary works, as well as treatises on ritual and grammar. Modern scholars of ancient India have noted the truly remarkable achievements of the Indian pandits who have preserved enormously bulky texts orally for millennia.

IV. STYLES OF MEMORIZATION

Prodigious energy was expended by ancient Indian culture in ensuring that these texts were transmitted from generation to generation with inordinate fidelity.For example, memorisation of the sacred Vedas included up to eleven forms of recitation of the same text. The texts were subsequently read-proof by comparing the different recited versions.

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V. THE WRITTEN TRADITION: PROSE COMMENTARY

With the increasing complexity of mathematics and other exact sciences, both writing and computation were required. Consequently, many mathematical works began to be written down in manuscripts that were then copied and re-copied from generation to generation.

India today is estimated to have about thirty million manuscripts, the largest body of handwritten reading material anywhere in the world. The literate culture of Indian science goes back to at least the fifth century B.C. ... as is shown by the elements of Mesopotamian omen literature and astronomy that entered India at that time and (were) definitely not ... preserved orally.

The earliest mathematical prose commentary was that on the work, Aryabhaṭtya (written 499 CE), a work on astronomy and mathematics. The mathematical portion of the Aryabhaṭtya was composed of 33 sutras (in verse form) consisting of mathematical statements or rules, but without any proofs.

VI. NUMERALS AND THE DECIMAL NUMBER SYSTEM

It is well known that the decimal place-value system in use today was first recorded in India, then transmitted to the Islamic world, and eventually to Europe.The Syrian bishop Severus Sebokht wrote in the mid-7th century CE about the "nine signs" of the Indians for expressing numbers. However, how, when, and where the first decimal place value system was invented is not so clear.

The earliest extant script used in India was the Kharoṣṭhī script used in the Gandhara culture of the north-west. It is thought to be of Aramaic origin and it was in use from the 4th century BCE to the 4th century CE. Almost contemporaneously, another script, the Brahmi script, appeared on much of the sub-continent, and would later become the foundation of many scripts of South Asia and South-east Asia. Both scripts had numeral symbols and numeral systems, which were initially not based on a place-value system.

The earliest surviving evidence of decimal place value numerals in India and southeast Asia is from the middle of the first millennium CE.

It has been hypothesized that the Indian decimal place value system was based on the symbols used on Chinese counting boards from as early as the middle of the first millennium BCE. According to (Plofker 2009), these counting boards, like the Indian counting pits, ..., had a decimal place value structure ... Indians may well have learned of these decimal place value rod numerals from Chinese Buddhist pilgrims or other travellers or they may have developed the concept independently from their earlier non-place-value system.

VII. BAKHSHALI MANUSCRIPT

The oldest extant mathematical manuscript in India is the Bakhshali Manuscript, a birch bark manuscript written in Buddhist hybrid Sanskritin the Sarada script, which was used in the NorthWestern region of the Indian subcontinent between the 8th and 12th centuries CE. The manuscript was discovered in 1881 by a farmer while digging in a stone enclosure in the village of Bakhshali, near Peshawar.

VIII. CLASSICAL PERIOD (400–1600)

This period is often known as the golden age of Indian Mathematics. This period saw mathematicians such as Aryabhata, Varahamihira, Brahmagupta, Bhaskara I, Mahavira, Bhaskara II, Madhava of Sangamagramaand NilakanthaSomayaji give broader and clearer shape to many branches of mathematics. Their contributions would spread to Asia, the Middle East, and eventually to Europe. Unlike Vedic mathematics, their works included both astronomical and mathematical contributions. In fact, mathematics of that period was included in the 'astral science' (jyotiḥsastra) and consisted of three sub-disciplines: mathematical sciences (gaṇita or tantra), horoscope astrology (hora or jataka) and divination (saṃhita). This tripartite division is seen in Varahamihira's 6th century compilation—Pancasiddhantika(literally panca, five, siddhanta, 'conclusion of deliberation', 575 CE)—of five earlier works, Surya Siddhanta, RomakaSiddhanta, PaulisaSiddhanta, VasishthaSiddhanta and PaitamahaSiddhanta, which were adaptations of still earlier works of Mesopotamian, Greek, Egyptian, Roman and Indian astronomy.

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IX. FIFTH AND SIXTH CENTURIES

9.1 Surya Siddhanta

Though its authorship is unknown, the Surya Siddhanta (400 CE) contains the roots of modern trigonometry. Because it contains many words of foreign origin, some authors consider that it was written under the influence of Mesopotamia and Greece.

This ancient text uses the following as trigonometric functions for the first time:

- Sine (Jya).
- Cosine (Kojya).
- Inverse sine (Otkramjya).

It also contains the earliest uses of:

- Tangent.
- Secant.

Later Indian mathematicians such as Aryabhata made references to this text.

9.2 Chhedi calendar

This Chhedi calendar (594) contains an early use of the modern place-valueHindu–Arabic numeral system now used universally.

X. ARYABHATA I

Aryabhata (476–550) wrote the Aryabhatiya. He described the important fundamental principles of mathematics in 332 shlokas. The treatise contained:

- Quadratic equations
- Trigonometry
- The value of π, correct to 4 decimal places.

Aryabhata also wrote the Arya Siddhanta, which is now lost. Aryabhata's contributions include:

Trigonometry:

- Introduced the trigonometric functions.
- Defined the sine (jya) as the modern relationship between half an angle and half a chord.
- Defined the trigonometrical ratio cosine (kojya).
- Defined the versine (utkrama-jya).
- Defined the inverse sine (otkramjya).
- Gave methods of calculating their approximate numerical values.
- Contains the earliest tables of sine, cosine and versine values, in 3.75° intervals from 0° to 90°, to 4 decimal places of accuracy.
- Contains the trigonometric formula $\sin(n + 1)x - \sin nx = \sin nx - \sin(n - 1)x - (1/225)\sin nx$.
- Spherical trigonometry.

Arithmetic:

- Continued fractions.

Algebra:

- Solutions of simultaneous quadratic equations.
- Whole number solutions of linear equations by a method equivalent to the modern method.
- General solution of the indeterminate linear equation.

Mathematical astronomy:

- Accurate calculations for astronomical constants, such as the:
    - Solar eclipse.
    - Lunar eclipse.
    - The formula for the sum of the cubes, which was an important step in the development of integral calculus.

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XI. VARAHAMIHIRA

Varahamihira (CE. 505–587) produced the PanchaSiddhanta (The Five Astronomical Canons). He made important contributions to trigonometry, including sine and cosine tables to 4 decimal places of accuracy and the following formulas relating sine and cosine functions:

In the 7th century, two separate fields, arithmetic (which included measurement) and algebra, began to emerge in Indian Mathematics. The two fields would later be called pati-gaṇita (literally Mathematics of algorithms) and bija-gaṇita (literallyMathematics of seedswith seeds—like the seeds of plants—representing unknowns with the potential to generate, in this case, the solutions of equations). Brahmagupta, in his astronomical work Brahma SphuṭaSiddhanta (628 CE), included two chapters (12 and 18) devoted to these fields. Later he gave the first explicit solution of the quadratic equation**.**

XII.BHASKARA I

Bhaskara I (600–680) expanded the work of Aryabhata in his books titled Mahabhaskariya, Aryabhatiya-bhashya and Laghu-bhaskariya. He produced:

- Solutions of indeterminate equations.
- A rational approximation of the sine function.
- A formula for calculating the sine of an acute angle without the use of a table, correct to two decimal places.

XIII.NINTH TO TWELFTH CENTURIES

13.1 Virasena

Virasena (8th century) was a Jain mathematician in the court of Rashtrakuta King Amoghavarsha of Manyakheta, Karnataka. He wrote the Dhavala, a commentary on Jain mathematics, which:

Deals with the concept of ardhaccheda, the number of times a number could be halved, and lists various rules involving this operation.

13.2 Mahavira

Mahavira Acharya (C. 800–870) from Karnataka, the last of the notable Jain mathematicians, lived in the 9th century and was patronised by the Rashtrakuta king Amoghavarsha. He wrote a book titled Ganit Saar Sangraha on numerical mathematics, and also wrote treatises about a wide range of mathematical topics. These include the mathematics of:

- Zero
- Squares
- Cubes
- square roots, cube roots, and the series extending beyond these
- Plane geometry
- Solid geometry
- Problems relating to the casting of shadows
- Formulae derived to calculate the area of an ellipse and quadrilateral inside a circle.

Mahavira also:

  - Solved indeterminate quadratic equations.
  - Solved indeterminate cubic equations.
  - Solved indeterminate higher order equations.
- Asserted that the square root of a negative number did not exist
- Gave the sum of a series whose terms are squares of an arithmetical progression, and gave empirical rules for area and perimeter of an ellipse.
- Solved cubic equations.
- Solved quartic equations.
- Solved some quintic equations and higher-order polynomials.
- Gave the general solutions of the higher order polynomial equations:

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13.3 Shridhara

Shridhara (C. 870–930), who lived in Bengal, wrote the books titled Nav Shatika, Tri Shatika and Pati Ganita. He gave:

- A good rule for finding the volume of a sphere.
- The formula for solving quadratic equations.

The Pati Ganita is a work on arithmetic and measurement. It deals with various operations, including:

- Elementary operations
- Extracting square and cube roots.
- Fractions.
- Eight rules given for operations involving zero.
- Methods of summation of different arithmetic and geometric series, which were to become standard references in later works.

13.4 Manjula

Aryabhata's differential equations were elaborated in the 10th century by Manjula (also Munjala).He understood the concept of differentiation after solving the differential equation that resulted from substituting this expression into Aryabhata's differential equation.

13.5 Aryabhata II

Aryabhata II (c. 920–1000) wrote a commentary on Shridhara, and an astronomical treatise Maha-Siddhanta. The Maha-Siddhanta has 18 chapters, and discusses:

- Numerical mathematics (AnkGanit).
- Algebra.
- Solutions of indeterminate equations (kuttaka).

13.6 Shripati

Shripati Mishra (1019–1066) wrote the books SiddhantaShekhara, a major work on astronomy in 19 chapters, and GanitTilaka, an incomplete arithmetical treatise in 125 verses based on a work by Shridhara. He worked mainly on:

- Permutations and combinations.
- General solution of the simultaneous indeterminate linear equation.

He was also the author of Dhikotidakarana, a work of twenty verses on:

- Solar eclipse.
- Lunar eclipse.

The Dhruvamanasa is a work of 105 verses on:

- Calculating planetary longitudes
- eclipses.
- planetary transits.

NemichandraSiddhantaChakravati
NemichandraSiddhantaChakravati (c. 1100) authored a mathematical treatise titled Gome-mat Saar.

13.7 Bhaskara II

Bhaskara II (1114–1185) was a mathematician-astronomer who wrote a number of important treatises, namely the Siddhanta Shiromani, Lilavati, Bijaganita, Gola Addhaya, GrihaGanitam and Karan Kautoohal. A number of his contributions were later transmitted to the Middle East and Europe. His contributions include:

Arithmetic:

- Interest computation
- Arithmetical and geometrical progressions
- Plane geometry
- Solid geometry
- The shadow of the gnomon
- Solutions of combinations
- Gave a proof for division by zero being infinity.

Algebra:

- The recognition of a positive number having two square roots.
- Surds.
- Operations with products of several unknowns.

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- The solutions of:
- Quadratic equations.
- Cubic equations.
- Quartic equations.
- Equations with more than one unknown.
- Quadratic equations with more than one unknown.
- The general form of Pell's equation using the chakravala method.
- The general indeterminate quadratic equation using the chakravala method.
- Indeterminate cubic equations.
- Indeterminate quartic equations.
- Indeterminate higher-order polynomial equations.

Geometry:

- Gave a proof of the Pythagorean theorem.

Calculus:

- Conceived of differential calculus.
- Discovered the derivative.
- Discovered the differential coefficient.
- Developed differentiation.
- Stated Rolle's theorem, a special case of the mean value theorem (one of the most important theorems of calculus and analysis).
- Derived the differential of the sine function.
- Computed π, correct to five decimal places.
- Calculated the length of the Earth's revolution around the Sun to 9 decimal places.

Trigonometry:

- Developments of spherical trigonometry

- The trigonometric formulas

XIV. KERALA MATHEMATICS (1300–1600)

The Kerala school of astronomy and mathematics was founded by Madhava of Sangamagrama in Kerala, South India and included among its members: Parameshvara, NeelakantaSomayaji, Jyeshtadeva, AchyutaPisharati, Melpathur Narayana Bhattathiri and Achyuta Panikkar. It flourished between the 14th and 16th centuries and the original discoveries of the school seems to have ended with Narayana Bhattathiri (1559–1632). In attempting to solve astronomical problems, the Kerala school astronomers independently created a number of important mathematics concepts. The most important results, series expansion for trigonometric functions, were given in Sanskrit verse in a book by Neelakanta called Tantrasangraha and a commentary on this work called Tantrasangraha-vakhya of unknown authorship. The theorems were stated without proof, but proofs for the series for sine, cosine, and inverse tangent were provided a century later in the work Yuktivasa(C.1500–C.1610), written in Malayalam, by Jyesthadeva.

Their discovery of these three important series expansions of calculus—several centuries before calculus was developed in Europe by Isaac Newton andGottfried Leibniz—was an achievement.

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